Rossmoyne Senior High School

Semester Two Examination, 2020

Question/Answer booklet

MATHEMATICS APPLICATIONS UNITS 3&4

Section One: Calculator-free

SOLUTIONS

WA student number: In figures | | | | | | | | |

In words _____

Teacher's name _____

Time allowed for this section

Reading time before commencing work: five minutes
Working time: fifty minutes

Number of additional answer booklets used (if applicable): | |

Materials required/recommended for this section

To be provided by the supervisor

This Question/Answer booklet
Formula sheet

To be provided by the candidate

Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: nil

Important note to candidates

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised material. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of examination
Section One: Calculator-free	8	8	50	52	35
Section Two: Calculator-assumed	13	13	100	98	65
				Total	100

Instructions to candidates

1. The rules for the conduct of examinations are detailed in the school handbook. Sitting this examination implies that you agree to abide by these rules.

2. Write your answers in this Question/Answer booklet preferably using a blue/black pen. Do not use erasable or gel pens.

3. You must be careful to confine your answers to the specific question asked and to follow any instructions that are specific to a particular question.

4. Show all your working clearly. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.

5. It is recommended that you do not use pencil, except in diagrams.

6. Supplementary pages for planning/continuing your answers to questions are provided at the end of this Question/Answer booklet. If you use these pages to continue an answer, indicate at the original answer where the answer is continued, i.e. give the page number.

7. The Formula sheet is not to be handed in with your Question/Answer booklet.

See next page

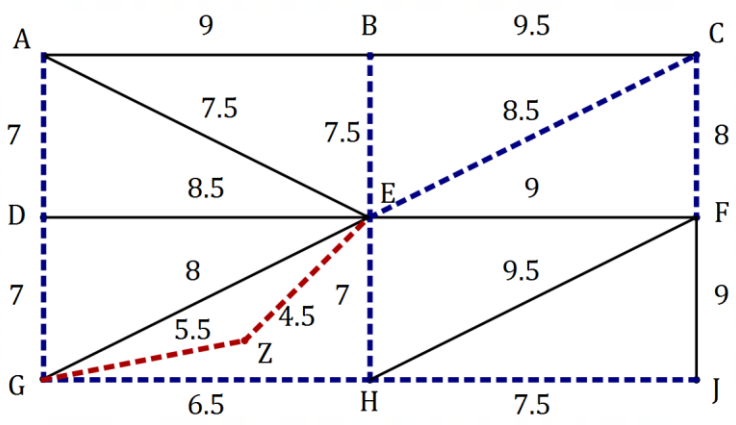
Section One: Calculator-free **35% (52 Marks)**

This section has **eight** questions. Answer **all** questions. Write your answers in the spaces provided.

Working time: 50 minutes.

Question 1 **(7 marks)**

The vertices on the graph below represent nine pumping stations. The edge weights are the times required to install new electrical cabling between the connected stations.



Solution (a)
See graph
Specific behaviours
✓ any spanning tree
✓ minimum spanning tree

Solution (c)(i)
See graph
Specific behaviours
✓ correctly adds edges, weights

(a) Clearly show the minimum spanning tree on the graph. (2 marks)

A contractor charges $\$100$ per hour to install the cabling.

(b) Determine the cost to install new electrical cabling using the minimum spanning tree. (2 marks)

Solution
$L = 7 + 7 + 6.5 + 7 + 7.5 + 7.5 + 8.5 + 8$ $= 59$ h $C = 100 \times 59$ $= \$5\,900$
Specific behaviours
✓ length of MST in hours
✓ correct cost

(c) A tenth pumping station Z is to be included. The time to install cable between Z and G is 5.5 hours and between Z and E is 4.5 hours.

(i) Use this information to add pumping station Z to the graph above. (1 mark)

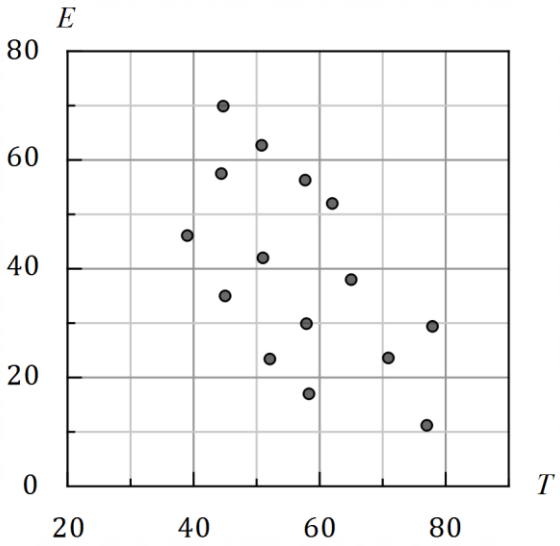
(ii) If the new cabling is now installed using the minimum spanning tree that includes Z, determine the extra cost of the installation. (2 marks)

Solution
Changes to minimum spanning tree: add GZ and ZE, drop EH. Change to length of minimum spanning tree is $5.5 + 4.5 - 7 = 3$. Hence extra cost is $3\times\ 100 = \$300$.
Specific behaviours
✓ indicates changes to minimum spanning tree
✓ correct extra cost

See next page

Question 2 **(5 marks)**

A sample of fifteen people were asked to take between 30 and 90 minutes to practice a new skill and then their percentage error score in performing the skill E recorded against their practice time T minutes. The results are shown on the scatterplot below.



(a) Describe the association between T and E in terms of direction, form and strength. (3 marks)

Solution
Direction is **negative**, form is **linear**, and strength is **moderate**.
Specific behaviours
✓ direction ✓ form ✓ strength

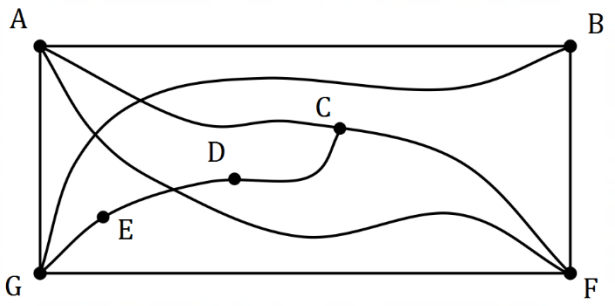
(b) Estimate, to one decimal place, the value of the correlation coefficient between the variables and hence determine the percentage of the variation in the error scores that can be explained by the variation in the practice times for this sample. (2 marks)

Solution
$r \approx -0.6 \Rightarrow r^2 = 0.36$ 36% of the variation in E can be explained by the variation in T. $FYI\ Classpad: E = -0.85T + 88.22 \quad r = -0.58 \quad r^2 = 0.34$
Specific behaviours
✓ estimate to 1dp between -0.4 and -0.8 inclusive ✓ correctly squares r and writes as percentage

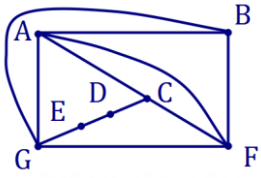
See next page

Question 3 **(7 marks)**

Graph M is shown at right.



(a) Adding missing vertices as necessary to those below, draw graph M in the plane, to clearly show that it is planar. (2 marks)

Solution
 *(Example only)*
Specific behaviours
✓ no edges that cross, at least 4 correct vertex degrees ✓ correctly drawn in the plane

(b) Show that graph M satisfies Euler's formula. (2 marks)

Solution
$v = 7, \quad e = 11, \quad f = 6$ $v + f - e = 7 + 6 - 11 = 2$
Specific behaviours
✓ correctly counts v, e, f ✓ substitutes into Euler's formula and simplifies

(c) Graph M is semi-Eulerian. Describe two features of the trail it must contain to be classified as semi-Eulerian. (2 marks)

Solution
The trail is open and includes every edge exactly once. (*Only accept 'trail starts and ends at odd degree vertices' instead of 'trail is open' if answer makes clear the odd vertices are different.*)
Specific behaviours
✓ indicates trail is open ✓ indicates trail includes every edge exactly once

(d) Describe where an edge can be added to graph M so that it contains an Eulerian trail. (1 mark)

Solution
Add edge between vertex C and vertex B.
Specific behaviours
✓ indicates correct vertices

See next page

Question 4 **(6 marks)**

Annie, Bob, Chris and Denise have been chosen for the 4×50 m medley relay team in a swimming carnival. A medley relay is swum by four different swimmers, each swimming one of four different strokes. Their best times, in seconds, to swim 50 m freestyle, backstroke, breaststroke and butterfly are shown in the following table.

	Freestyle	Backstroke	Breaststroke	Butterfly
Annie	33	43	37	38
Bob	33	44	37	37
Chris	34	43	37	38
Denise	34	42	38	37

(a) Show use of the Hungarian algorithm to determine which stroke each student should swim so that the team completes the 4×50 m medley relay in the shortest possible time. (4 marks)

Solution

$$\begin{bmatrix} 0 & 10 & 4 & 5 \\ 0 & 11 & 4 & 4 \\ 0 & 9 & 3 & 4 \\ 0 & 8 & 4 & 3 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 2 & 1 & 2 \\ 0 & 3 & 1 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 1 & 0 & 1 \\ 0 & 2 & 0 & 0 \\ 1 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \end{bmatrix} \text{ or } \begin{bmatrix} 0 & 1 & 1 & 1 \\ 0 & 2 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 2 & 0 \end{bmatrix}$$

Assignment: Annie - Freestyle; Bob - Butterfly;
Chris - Breaststroke; Denise - Backstroke.

Specific behaviours

✓ reduces rows
✓ reduces columns
✓ reduces again so that four lines needed to cover
✓ states assignment

(b) The record for the 4×50 m medley relay is 2 minutes and 31 seconds. If all the students swim their assigned leg in their best time, will they break the record? Justify your answer. (2 marks)

Solution

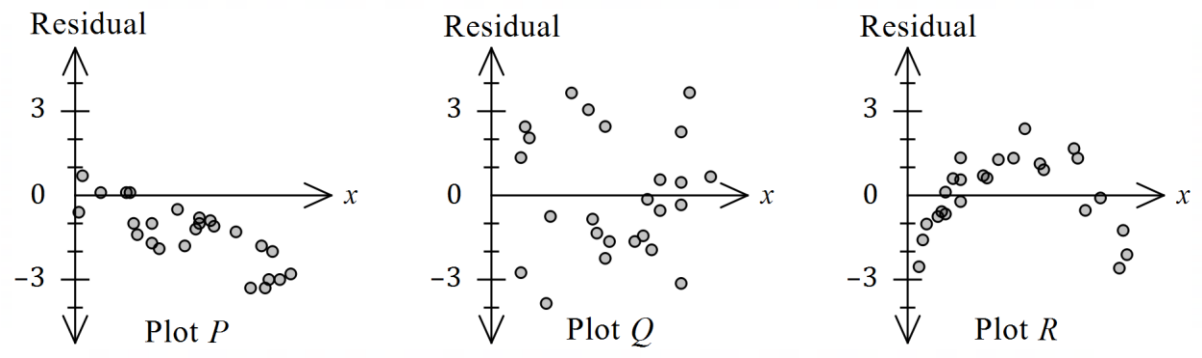
Yes. Their time will be $33 + 37 + 37 + 42 = 149$ seconds and the record is 151 seconds, which is 2 second slower.

Specific behaviours

✓ indicates time for students
✓ compares to record and states yes

Question 5 **(6 marks)**

A linear model was fitted to datasets A, B and C and the resulting residual plot for each model shown below. Dataset A has a non-linear form whereas datasets B and C have linear form. It is also known that the linear, using a line of best fit, was incorrectly fitted to dataset B.



(a) State, with justification, which residual plot is most likely to be derived from dataset A. (2 marks)

Solution
Plot R: A pattern is evident in the residual plot. (As x increases, the residuals tend to be negative and then positive and then negative again.)
Specific behaviours
✓ graph ✓ justification

(b) State, with justification, which residual plot is most likely to be derived from dataset B. (2 marks)

Solution
Plot P: Almost all the residuals are negative, which is unusual and suggests an incorrectly fitted linear model.
Specific behaviours
✓ graph ✓ justification

(c) State, with justification, which residual plot is most likely to be derived from dataset C. (2 marks)

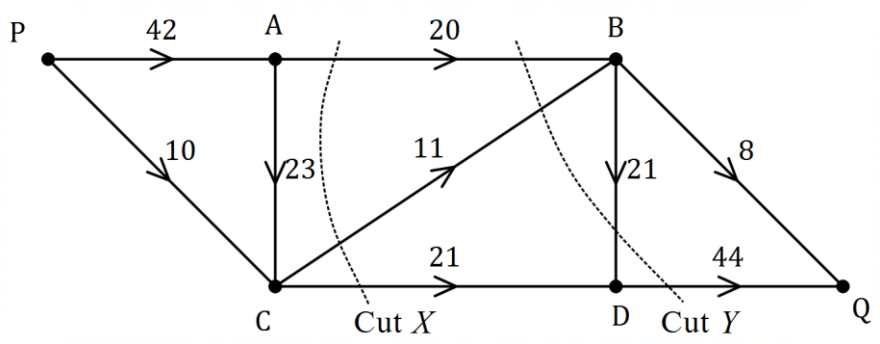
Solution
Plot Q: There is no pattern evident in the residuals.
Specific behaviours
✓ graph ✓ justification

Note
For one graph (correctly matched), accept the logical justification that it is the only dataset left - if that is the case.

See next page

Question 6 **(8 marks)**

A directed subgraph of a distribution network is shown below. The vertices represent distribution centres and the weight on each edge is the maximum volume of parcels, in cubic metres, that can be transported from one distribution centre to another every day.



(a) Determine the capacity of cut X and the capacity of cut Y shown above. (2 marks)

Solution
Cut X is $20 + 11 + 21 = 52$ m^3. Cut Y: $20 + 11 + \cancel{(-21)} + 44 = 75$ m^3.
Specific behaviours
✓ cut X; ✓ cut Y

(b) Determine the maximum volume of parcels that can be transported

(i) from centre P to centre C in a day. (1 mark)

Solution
Maximum volume is 33 m^3.
Specific behaviours
✓ correct maximum flow

(ii) from centre C to centre D in 2 days. (2 marks)

Solution
Minimum cut is 32 and so maximum volume in 2 days is $32 \times 2 = 64$ m^3.
Specific behaviours
✓ minimum cut; ✓ maximum volume

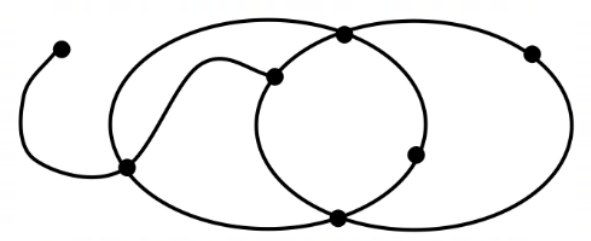
(c) Determine the maximum volume of parcels that can be transported from centre P to centre Q in 5 days. (3 marks)

Solution
Cut $PA, PC = 44$, cut $BQ, DQ = 43$, cut $AB, AD, AC, PC = 50$ and so minimum cut is 50. Maximum volume in 5 days is $5 \times 50 = 250$ m^3 of parcels.
Specific behaviours
✓ at least two more cuts ✓ indicates minimum cut ✓ correct volume of parcels

Alternative Solution
$PABQ = 8$, $PABDQ = 12$, $PACBDQ = 9$, $PACDQ = 13$, $PCDQ = 8$. Maximum flow is 50. Maximum volume in 5 days is $5 \times 50 = 250$ m^3 of parcels.
Specific behaviours
✓ systematically lists flows ✓ correct maximum flow ✓ correct volume of parcels

See next page

Question 7 **(6 marks)**



Graph G is shown above. It represents a network of tracks between seven camp sites.

(a) State, with reasons, if graph G is a simple graph. (2 marks)

Solution
Yes - there are no loops or multiple edges.
Specific behaviours
✓ states yes with an some explanation ✓ correct explanation

(b) For graph G, determine the length of the longest

(i) closed path it contains. (1 mark)

Solution
Length is 5.
Specific behaviours
✓ correct length

(ii) open trail it contains. (1 mark)

Solution
Length is 10.
Specific behaviours
✓ correct length

(c) Explain why graph G is a semi-Hamiltonian graph. (2 marks)

Solution
Graph G contains an open path (but not a cycle) that includes each vertex in the graph.
Specific behaviours
✓ indicates open path ✓ indicates path includes each vertex

End of questions

Question 8 **(7 marks)**

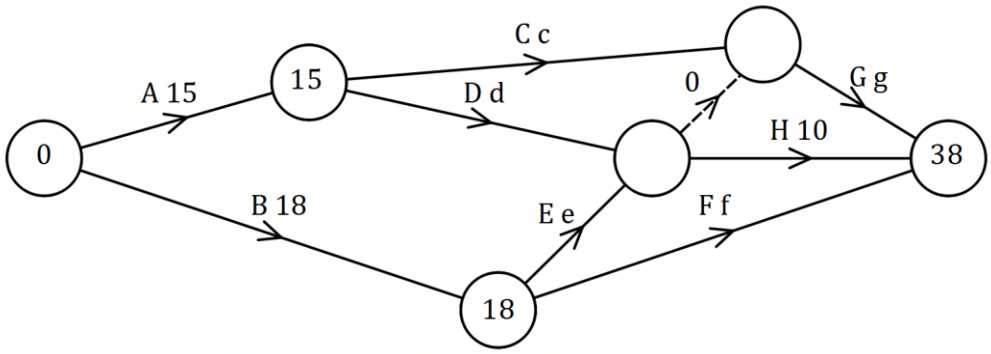
A project involves the completion of activities A to H, as shown in the following table. Note that only three of the activity durations are shown.

Activity	A	B	C	D	E	F	G	H
Immediate predecessor(s)	–	–	A	A	B	B	C, D, E	D, E
Duration (weeks)	15	18						10
Float (weeks)	0	3	8	0	4	3	3	0

The minimum completion time for the project is 38 weeks.

(a) Construct an activity network to represent the above information. (3 marks)

Solution



Specific behaviours

✓ shows most activities as labelled edges
✓ correct predecessors for each activity
✓ correct activity network including dummy edge, with direction

(c) List the tasks that lie on the critical path. (1 mark)

Solution

The tasks are A, D and H (zero float).

Specific behaviours

✓ correct tasks

(b) Determine a possible duration for each of the activities C, D, E, F and G. (3 marks)

Solution

Using critical path: $d = 38 - 15 - 10 = 13$

Using Duration=LST of next−EST−Float:

$LST_{G,H} = 38 - 10 = 28, \quad g = 38 - 28 - 3 = 7, \quad c = (38 - 7) - 15 - 8 = 8$

$e = 28 - 18 - 4 = 6, \quad f = 38 - 18 - 3 = 17$

Hence durations of C, D, E, F and G are $8, 13, 6, 17$ and 7 weeks respectively.

Specific behaviours

✓ duration of D
✓ durations of C and G
✓ durations of E and F

Supplementary page

Question number: _____

End of questions